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# Decoupling The Role Of Inertia And Gravity On Particle Dispersion

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## Abstract

Turbulent gas flows laden with small, dense particles are encountered in a wide number of important applications in both industrial settings and aerodynamics applications. Particle interactions with the underlying turbulent flow are exceedingly complex and, consequently, difficult to accurately model. The difficulty arises primarily due to the fact that response of a particle to the local environment is dictated by turbulence properties in the reference frame moving with the particle (particle-Lagrangian). The particle-Lagrangian reference frame is in turn dependent upon the particle relaxation time (time constant) as well as gravitational drift. The combination of inertial and gravitational effects in this frame complicates our ability to accurately predict particle-laden flows since measurements in the particle-Lagrangian reference frame are difficult to obtain.

Therefore, in this work we will examine separately the effects of inertia and gravitational drift on particle dispersion through a combination of physical and numerical experiments. In this study, particle-Lagrangian measurements will be obtained in physical experiments using stereo image velocimetry. Gravitational drift will be varied in the variable-g environments of the NASA DC-9 and in the zero-g environment at the drop tower at NASA-Lewis. Direct numerical simulations will be used to corroborate the measurements from the variable-g experiments. We expect that this work will generate new insight into the underlying physics of particle dispersion and will, in turn, lead to more accurate models of particle transport in turbulent flows.

## Introduction

In modeling particle-laden flow, one must have a fundamental understanding of how the particle responds to local turbulence. Defining this turbulence is probably the greatest difficulty in deriving an accurate model. The difficulty arises from the fact that each particle navigates a unique path through the flow: a path dictated by its inertia and gravitational drift. This path, dubbed the particle-Lagrangian reference frame, is neither the purely Lagrangian path of a fluid point, nor the conventional stationary Eulerian reference frame. Particle inertia affects the turbulence experienced by the particle as particles are flung from one fluid neighborhood to another, and gravity affects the path

by pulling the particle through the turbulence. To accurately model the behavior of the particle, one must decouple the effects of the gravitational drift from those resulting from the inertia of the particle: this is the goal of the proposed research.

If one imagines a turbulent fluid field consisting of a random assortment of various size eddies, then the turbulence can be partially characterized by a power spectrum: a measure of the distribution of the turbulent kinetic energy among these eddies. The behavior of an individual particle will depend on how quickly a particle can respond to these fluctuations in the fluid velocity. For instance, small (high frequency) eddies will have little effect

on particles with slow response times (large time constants) and, conversely, large (low frequency) eddies will have little difficulty in influencing all but the most sluggish particles. The apparent frequency of the eddy experienced by the particle, however, will be a function of the particle velocity. Similar to an acoustic Doppler shift, as the particle moves through an eddy, the frequency it responds to will be a function of the particle velocity as well. Finally, to further complicate modeling, as it crosses the eddies due to its gravitational drift, it moves from one fluid neighborhood to another. This is known as the “crossing-trajectories” effect<sup>1</sup>, and plays an active role in the particle dispersion<sup>2</sup>.

#### Previous Work

Although there has been a substantial effort to better understand and model the transport of a dilute particle species in a turbulent field<sup>3,4</sup>, accurate modeling has been hindered by the inability to transform the Eulerian (or Lagrangian) fluid models into the particle-Lagrangian reference frame. Csanady<sup>5</sup> assumed a simplified particle transport equation

$$\frac{d\vec{V}_p}{dt} = \frac{1}{\tau_p} (\vec{U}_f - \vec{V}_p) - \vec{g} \quad [1]$$

where  $U_f$  and  $V_p$  are the fluid and particle velocities and  $\tau_p$  is the particle time constant. This equation is the transport equation for particles whose density is much greater than that of the carrier fluid. The particle time constant is a measure of the response time and for Stokesian particles is

$$\tau_p = \frac{\rho d^2}{18\mu} \quad [2]$$

where  $\rho$  and  $d_p$  are the particle density and diameter,

respectively, and  $\mu$  is the viscosity of the fluid. Finally, a Stokesian particle is one whose Reynolds number is less than 0.1. The particle Reynolds number is simply defined as

$$Re_p = \frac{U_s d}{\nu} \quad [3]$$

where  $U_f$  is the fluid velocity relative to the particle and  $\nu$  is the fluid kinematic viscosity.

From this transport equation, Csanady showed that if one examined the fluctuating component of equation 1 (Reynolds decomposition) and replaced the particle and fluid velocity by a Fourier series, one could estimate the fluid power spectrum by

$$E_p(\nu) = \frac{1}{1 + 4\pi^2 \tau_p^2 \nu^2} E_f(\nu) \quad [4]$$

where  $E_p$  and  $E_f$  are the particle and fluid power spectra,  $\nu$  is the eddy frequency (in Hz), and  $\tau_p$  is the particle time constant.<sup>6</sup> Csanady assumed that  $E_f$  is the fluid power spectrum in the particle-Lagrangian coordinate system. If one could accurately transform measurements made in the Eulerian reference frame to the particle-Lagrangian, one could then use the above analysis to estimate the particle response - and - from there - the particle autocorrelation. Once one has the particle autocorrelation, one can then predict the particle dispersion ( $y_2(t)^2$ ) with a Taylor-like analysis<sup>7</sup>:

$$\overline{y_2(t)^2} = 2\overline{v_2^2} \int_0^t \int_0^\eta R_L(\tau) d\tau d\eta \quad [5]$$

where  $R_L(\tau)$  is the particle velocity autocorrelation and  $v_2^2$  is the average square particle velocity fluctuation for homogeneous stationary turbulence.

To date, there have been three approaches to trying

to decouple gravity and inertia to model this transformation. The first is by charging the particles and then using an electric field to essentially remove the effects of gravity (the Wells and Stock experiment<sup>2</sup>), the second is through numerical simulations, and the third is a combination of the two.

#### The Wells and Stock Experiment

Wells and Stock were able to make similar measurements to the ones proposed here by electrically charging the particles and using a electric field to vary the particle drift velocity. They introduced electrically charged two different diameter glass beads (5 and 57  $\mu\text{m}$ ) into a near-homogeneous flow. They found that the effect crossing-trajectories on particle dispersion was negligible for particles with drift velocities that were less than the r.m.s. velocity of the fluid. This would correspond to particles that almost follow the fluid and therefore will not experience the crossing trajectories effect. Particles with drift velocities equal to the r.m.s. velocity of the fluid reduced the dispersion coefficient by about 10%. They had some difficulty, however, with particle charge and shape non-uniformity affecting their results. This, in turn, led to a difficulty in making conclusions on the effects of inertia.

They did, however, demonstrate that the crossing trajectories effect reduces particle dispersion. The dispersion decreases because as the particle moves through fluid neighborhoods, the fluid fluctuations experienced by the particle lose correlation more rapidly and therefore (from equation 4) reduce the

particle velocity correlation and hence a decrease in dispersion (equation 5). They were also able to show that the magnitude of the gravitational drift directly affects the particle behavior in a non-linear way.

Unfortunately, in their work they were only able to examine two particles and had difficulties in retaining uniform charge on the particles. In the future experiments, we would first extend their work in two ways: (1) examine a greater range of particle time constants and (2) examine the effect of inertia on particle dispersion. The latter could not be accurately ascertained using their experimental setup. The last stage of the work would be to correlate instantaneous particle behavior to the instantaneous fluid behavior, making it easier to extend the results to a wider class of applications.

#### Numerical Work

As mentioned above, the majority of "purely numerical" studies of particle dispersion in turbulence have been performed using direct numerical simulation. In DNS the Navier-Stokes equations are solved without approximation (other than those associated with the numerical method) and the results from a DNS calculation may be analyzed in much the same fashion as measurements from a laboratory experiment. The main disadvantage of DNS is that it remains limited to calculation of moderate Reynolds number canonical flows. However, given the extremely detailed description of the flow in a DNS computation, it provides a powerful tool for simulation and analysis.

Several investigators have used DNS to examine particle transport in isotropic turbulence (e.g., see Squires and Eaton<sup>8</sup>, Elghobashi and Truesdell<sup>9</sup>, Wang and Maxey<sup>10</sup>). Each of these investigations have demonstrated the utility of DNS for examination of fundamental aspects of particle dispersion in canonical flows. Squires and Eaton found good agreement between particle dispersion in DNS and the theory of Csanady<sup>5</sup>. Elghobashi and Truesdell also obtained good agreement between DNS predictions and the experimental measurements of Snyder and Lumley<sup>11</sup>. Wang and Maxey demonstrated that increases in particle settling velocities obey Kolmogorov scaling. While each of these previous efforts are relevant to this study, the principal aim of the DNS calculations proposed in this work is to corroborate the measurements obtained from the variable-g measurements. The parameter combinations required in the proposed simulations are not directly available from previous work.

### **Plan of Action**

In this work we will decouple the effects of gravity and inertia in particle dispersion. With a better understanding of how one can estimate the fluid turbulence in the reference frame of a moving particle, one can better estimate the particle response to its environment. The work will be divided into six segments: (1) development of the laboratory experiment, (2) computer simulations of the expected particle behavior for variable-g, (3) quasi-simulations tracking an imaginary particle through the measured fluid velocity field, (4) experimental

tests on a local aircraft for small time durations to verify and hone the experimental techniques, (5) variable-g experiments in the NASA DC-9 and drop tower, and (6) final data analysis and model development. First we will develop, build and test a sample experimental setup at Tufts - testing out the stereo-imaging velocimetry system (SIV) and refining the particle/fluid discrimination system. In tandem, we will run simulations of our flow at low Reynolds numbers. We will compare the simulation results with the measurements made in the laboratory. Next, using a local airplane, we will test the experiment in a near-zero-g environment. Combining these results, we will then run a series of experiments in the NASA DC-9 at variable-g. We will compare these results to both the full simulations and the quasi-simulations, and to the results of Wells and Stock. These final data will serve two purposes; (1) be the basis for our modeling effort and (2) quantify the accuracy of our quasi-numerical scheme, from which we can numerically examine a large number of particle parameters.

### **Experiment Design: Ground-Based: Laboratory**

#### **Experimental**

The experimental apparatus will be a simple suction tunnel with a variable-sized entrance grid to generate the various turbulence scales (see figure 1). The idea is to track particles in three dimensions through the simplest turbulent flow. This tunnel design is based on Corrsin's<sup>13</sup> and Pope's work<sup>14</sup> and should provide a relatively homogeneous turbulent flow. The honeycomb at the exit reduces the effects of the fan blades propagating upstream and the particle injector at the tunnel entrance allows us to

control the particle initial conditions. Turbulence properties (such as the energy spectrum) will also be measured at the injection location and will therefore permit comparison to simulations. The total height of the tunnel should be about 2 meters - this allows for roughly 20 particle time constants in the test section and another 10 for the entrance.

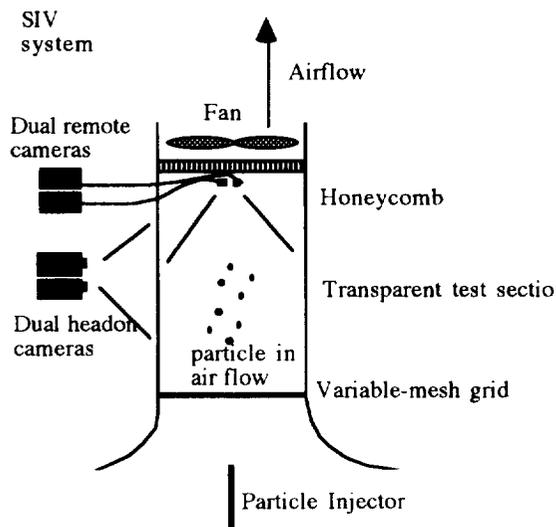


Figure 1: Experiment Schematic

The particle injection system for the ground-based experiment will be a single injection vacuum system so that we can control the particle injection time and speed to accurately compare with simulation results. For the flight experiments, we will use a continuous particle injection system that will allow us to track numerous particles in a given run (due to the time limitations of the flight experiments). We will distinguish particles from fluid tracers by using fluorescent particles (we can therefore discriminate based on reflected color). The air will be seeded with micron-size particles for the stereo image velocimetry (SIV) system using a simple paint-sprayer and talcum powder suspended

in alcohol. This seeding technique has been successfully used for seeding for LDV measurements - it relies on the alcohol evaporating before the talcum powder enters the test section.

*Experiment Design: Numerical*

The proposed simulations of particle-laden isotropic turbulence are direct numerical simulations. DNS of the incompressible Navier-Stokes equations shall be performed using the pseudo-spectral method of Rogallo (1981). In this method the dependent variables are represented using Fourier series expansions. Aliasing errors are eliminated using a combination of coordinate shifts and truncation. The discrete system of equations is time advanced using second-order Runge Kutta.

Treatment of a dispersed second phase of heavy particles is performed through numerical integration of the equation of motion for a large ensemble of particles. For particles with material densities much larger than the surrounding carrier flow the most significant forces governing motion are drag and gravity. The code uses equation 1 to describe the motion of a particle. Advancement of (1) requires the fluid velocity along the particle trajectory. Since it is unlikely particles are located at grid points where the turbulence velocity is available, calculation of the drag force requires interpolation of the fluid velocity from the grid to the instantaneous particle position. A recently developed B-spline method will be used for velocity interpolation.

The material properties of the particles will be identical to those used in the experiments. Previous investigations have shown that the sample size

necessary for adequate convergence of statistical quantities such as particle mean-square dispersion and the velocity autocorrelations is approximately 4,000. Similar sample sizes will be used in this work.

### **Conclusions**

In conclusion, the work we will be performing will look at how particle inertia gravity independently affect its motion. An accurate understanding of these effects will allow for accurate modeling of particle motion in a large number of industrial flows, including those being performed in zero-g environments. Work on this project will begin June 1.

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